Convex Optimization Approach to Observer-Based Stabilization of Linear Systems with Parameter Uncertainties

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Abstract—In this paper we investigate the design of observer-based controller for uncertain linear systems. On the basis of the approach using the Lyapunov theory jointly with linear matrix inequalities (LMIs), and by handling judiciously the Young relation, we derive new sufficient linear matrix inequality (LMI) conditions for the asymptotic stabilizability. The proposed method allows to compute simultaneously the observer and controller gains by solving only one LMI. The developed approach is then extended to both continuous-time systems with parameter uncertainties and their Euler approximation models. We show that our approach contains, as a particular solution, the elegant results established in [1]. A numerical example is provided to compare with respect to some existing methods.

I. INTRODUCTION

Observer design for both discrete-time and continuous time nonlinear systems has been the subject of numerous research contributions (see, e.g. [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]). However, a little attention has been dedicated to observer-based control in the uncertain linear case (see, e.g. [1], [2], [12], [13], [14], [4], [5], [15], and still an open problem (see, e.g. [16]). It is worthwhile to mention that the main difficulty that encountered in adressing the observer-based controller for uncertain linear is a non-convex problem (see, e.g. [12], [17]). Beside this obstacle, the observer-based feedback must be conceived not only to stabilize the system but also to deal with the impact of eventual uncertainties. In such a framework, the available results contain matched uncertainties [15], norm-bounded uncertainties [2], [1], [12], [18], [19], [11], and uncertainties of dyadics types [20]. This paper focuses on the problem of the observer-based control problem for a system subjected to norm-bounded uncertainties.

Our goal is to propose an LMI approach to adress the observer based problem for both discrete-time and continuous time uncertain linear systems. We will focus our study on papers that have examined the same problem, with LMI approach [21], rather than on papers which deal with this problem via other approaches (see, e.g. [22] and [23]). The LMI condition for the stabilization of nonlinear discrete-time systems and Euler approximation models has been extensively investigate in [2], [1], [9], [10], [11]. In comparison with existing results in this field, especially those in [2], we propose, in this paper, new sufficient LMIs that guarantee the stability of discrete-time uncertain linear systems under the action of observer-based feedbacks. It will be shown that the computation of observer and controller gains can be done simultaneously, by solving only one LMI. We first reduce the conservatism introduced by the two steps computing done in [2]. We then applied our approach to deduce a design of dynamic output feedback controllers through Euler approximate models : uncertain linear systems, we derive systematically from our first approach new less restrictive LMI condition compared to that in [1]. Finally, we show the usefulness of our method through an illustrative example. Our method is, first, based on Lyapunov theory which leads to stabilization conditions, in BMI setting. Then, using some judicious algebraic transformations on the bilinear matrix inequality (BMI) containing the observer and the controller gains, we give new completely LMI conditions for stabilization of uncertain linear systems.

The following notations will be used throughout this paper. The notation (*) is used for the blocks induced by symmetry; \( A^T \) represents the transposed matrix of \( A \); \( \mathbb{R}^{n \times m} \) is the set of all real \( n \) by \( m \) matrices; \( I \) is an identity matrix with approximate dimension; the value 0 denotes a zero matrix with approximate dimension; for a square matrix \( S \), \( S > 0 \) (\( S < 0 \)) means that this matrix is positive definite (negative definite); and \( A < B \) means that the matrix \( B - A \) is symmetric positive semi definite.

Before tackling the main issues of this paper, let us recall some known results that are used in the statement proofs later :

**Lemma 1.1 (Young’s inequality):** For given matrices \( X \) and \( Y \) with appropriate dimensions, we have for any invertible matrix \( S \) and scalar \( \varepsilon > 0 \),

\[
X^T Y + Y^T X \leq \varepsilon X^T S X + \frac{1}{\varepsilon} Y^T S^{-1} Y. \tag{1}
\]

The schur Lemma is needed to deduce an LMI feasible problem.

**Lemma 1.2 (Schur Lemma):** Let \( Q_1, Q_2 \) and \( Q_3 \) be three matrices of appropriate dimensions such that \( Q_1 = Q_1^T \) and \( Q_3 = Q_3^T \). Then, \( Q_3 < 0 \) and \( Q_1 - Q_2 Q_3^{-1} Q_2^T < 0 \) if and only if

\[
\begin{bmatrix}
Q_1 & Q_2 \\
Q_2^T & Q_3
\end{bmatrix} < 0.
\]