- Les conditions de 1er ordre :

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial x} = X y^{\frac{1}{2}} - 40\lambda = 0 \\ \dfrac{\partial \mathcal{L}}{\partial y} = \frac{1}{4} X^2 y^{-1/2} - 20\lambda = 0 \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} = 1600 - 20X - 40Y = 0 \end{cases} \qquad \begin{cases} \lambda = \dfrac{X Y^{\frac{1}{2}}}{40} \dots (1) \\ \lambda = \dfrac{\frac{1}{4} X^2 Y^{-1/2}}{20} \dots (2) \\ 1600 - 20X - 40Y = 0 \dots (3) \end{cases}$$

2,5

$$(1) = (2) \Rightarrow \frac{X Y^{1/2}}{40} = \frac{1/4 X^2 Y^{-1/2}}{20}$$

Aprés calcul : $Y = 0,5X \dots (4)$

On remplace dans (3) :

$$1600 - 40X - 20(0,5X) = 0 \Rightarrow 1600 = 50X$$

$$X = \frac{1600}{50} \Rightarrow \begin{array}{l} X = 32 \\ Y = 16 \end{array}$$

0,5

0,5

$\boxed{U = 2048}$

0,5

(3) TMS xy à l'équilibre :

$$TMS_{xy} = \frac{P_x}{P_y} = \frac{40}{20}$$

$$\boxed{TMS_{xy} = 2}$$

0,5

$$TMS_{xy} = \frac{UMX}{UMY} = \frac{X Y^{1/2}}{\frac{1}{4} X^2 Y^{-1/2}} = \frac{Y}{\frac{1}{4} X}$$

$$\boxed{TMS_{xy} = 2}$$

0,5

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